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Project Proposal

EE 109: Convex Optimization

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**Quantity Prediction for Multi-Period Trading Strategies via Convex Optimization**

In his 1952 seminal paper “Portfolio selection”[[1]](#endnote-1), Harry Markowitz was the first to mathematically formulate the choice of investment portfolio as an optimization problem, trading off risk and return. The concepts developed in his paper on portfolio selection have paved the way for massive developments in the analysis of financial systems and financial decision making[[2]](#endnote-2)[[3]](#endnote-3).

The essence of portfolio optimization is in maximizing the returns of a portfolio of stocks (pieces of ownership in a publicly traded company) or other assets while minimizing the risk of the portfolio (based on the variance of the return). The return is measured by the increase in the price of the assets composing a portfolio relative to a prior time period. The general trend in economics is that as the forecasted return of an asset increases, the risk profile for the asset increases. When this is applied across an entire portfolio or collection of assets, the relationship between the assets dramatically affects the overall risk-return profile of the entire portfolio. E.g. when the value of oil increases, the value of oil refinery companies may also increase, while the value of transportation companies may decrease. The objective of portfolio optimization can be restated as selecting which and how many assets to purchase in order to minimize the risk-return profile (risk/return) of the entire selection.

The main significance of the theory has been in developing algorithms to trade assets to optimize portfolio selection. A major class of algorithms has been developed to utilize convex optimization to maximize returns while minimizing risk. The primary focus of these trading algorithms has been on single period optimization (SPO)[[4]](#endnote-4), in other words, minimize the risk-return profile of a portfolio over a single time period (minutes, hours, days, weeks, years, etc.). In recent years this work has transitioned into the emerging field of multi-period portfolio optimization or minimizing the risk-return profile over multiple time periods[[5]](#endnote-5). The advantage to this method is that the cost of doing each trade can be analyzed and taken into account by the model in future time periods rather than only the current time period.

To summarize in layman’s terms, the single period optimization tells the user what and how much of each asset to purchase, while the multi-period optimization provides some additional insight into when the assets should be purchased. One of the major limitations of this approach is that the markets are not perfect systems as they are modeled[[6]](#endnote-6). In order to purchase an asset someone else must be willing to sell the same asset to you. The previously described portfolio optimization methods assume that all assets can be bought or sold at the same price[[7]](#endnote-7), and same transaction cost, however, in reality these assets may need to be purchased in a variety of quantities at different prices. This case has been studied in single period optimization[[8]](#endnote-8)[[9]](#endnote-9)[[10]](#endnote-10). In this project I plan on extending these methods to the multi-period optimization case.

Key reference papers are in the endnotes below.

1. H. Markowitz. Portfolio Selection. *Journal of Finance*, 7(1):77-91, 1952. [↑](#endnote-ref-1)
2. P. Kolm et al. 60 Years of Portfolio Optimization. *European Journal of Operational Research*, 234:356-371, 2014. [↑](#endnote-ref-2)
3. R. Mansini, et al. 20 Years of Linear Programming Based Portfolio Optimization. *European Journal of Operational Research, 234:518-535, 2014.* [↑](#endnote-ref-3)
4. C. Fulga, B. Pop. Single period portfolio optimization with fuzzy transaction costs. *20th EURO Mini Conference “Continuous Optimization and Knowledge Based Techniques”.* 2008. [↑](#endnote-ref-4)
5. S. Boyd, et al. Multi-Period Trading via Convex Optimization. *Publications and Trends in Optimization,* 3(1):1-76, 2016. [↑](#endnote-ref-5)
6. S. Boyd, et al. Performance Bounds and Suboptimal Policies for Multi-Period Investment. *Foundations and Trends in Optimization,* 1(1):1-72, 2014. [↑](#endnote-ref-6)
7. D. Goldsmith. Transaction Costs and the Theory of Portfolio Selection. *The Journal of Finance,* 31(4):1127-1139, 1976. [↑](#endnote-ref-7)
8. R. Zhang, et al. Dynamic Portfolio Optimization with Liquidity Cost and Market Impact: A Simulation-and-Regression Approach. <https://arxiv.org/pdf/1610.07694.pdf>, 2017. [↑](#endnote-ref-8)
9. J. Abernathy, et al. Efficient Market Making via Convex Optimization, and a Connection to Online Learning. *ACM Transaction on Economics and Computation,* 1(2):12, 2013. [↑](#endnote-ref-9)
10. J. Abernathy, et al. An Optimization-Based Framework for Automated Market-Making. [*Arxiv Preprint*](https://arxiv.org/abs/1011.1941)*, 2010.* [↑](#endnote-ref-10)